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A SINGLE UNIVERSAL DIMENSIONLESS CURVE FOR GRAPHICAL SOLUTION OF FRICTION FACTOR IN THE IMPLICIT COLEBROOK-WHITE EQUATION

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ABSTRACT

The determination of friction factor is an important exercise in pipe flow problems involving computation of head losses. This paper presents a single universal curve for the graphical determination of friction factor that is based on the Colebrook-White Equation, and by taking the Reynold's Number and pipe relative roughness values as variables. The method is a significant improvement over the Moody Diagram in terms of its precision. The Moody diagram does not cover the entire continuum of relative roughness values and is plotted as discrete number of curves. Application of the proposed graphical procedure over the broader range of transition to turbulent region indicated that the mean percentage error of estimation is 0.36% and the maximum error is 2.3%. This result places the proposed graphical method in the category of very accurate to extremely accurate method.

Keywords: Friction factor, head loss, pipe roughness, moody diagram, turbulent flow.

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INTRODUCTION

Fluid flow problems in pipes in which Newton's Law of viscosity applies appear broadly in engineering, medical, science and other fields. The Darcy-Weisbach Equation along with the Moody diagram (Figure 1) have been commonly used to calculate head losses in pipes and closed conduits. The formula is also used in pipe network simulations in hydraulic calculations that employ continuity equation, conservation of energy and minor head loss calculations (Brown, 2002). The Darcy-Weisbach formula is given by:

$$H_L = f \cdot \frac{L}{D} \frac{V^2}{2g} \qquad \textbf{eqn 1}$$

In Equation (1), H_L is the energy loss in meters (SI units), f is the friction factor, L is the length of pipe, D the internal diameter of pipes, V is the average velocity of flow in pipe and g is acceleration due to gravity (9.81 m/s^2 in SI units). The friction factor has been the subject of extensive experiments over the years. The head loss relation with pipe diameter, velocity and viscosity in laminar flow was independently discovered by Hagen and Poiseuille. It was established as a result that the head loss varies linearly with velocity in the laminar range, a fact that was also observed and agreed by Darcy (1857). Friction loss in turbulent flows was experimented by a number of researchers including Prandtl, von Karman, Nikuradse, Darcy, Colebrook and others in which they carried out experiments over a wider range of Reynold's Number and pipe wall roughness values. In the fully turbulent region, the friction factor is mainly dependent on the pipe roughness and the formula given by Equation (2) was proposed by Nikuradse for such flow regime (Zeghadnia *et al*., 2019);

$$f = -2\log\left(\frac{\varepsilon/D}{3.7}\right)^{-2} \qquad \textbf{eqn 2}$$

In Equation (2), f is the pipe friction factor, ε is the pipe equivalent average roughness height and D is the pipe internal diameter. The transitional flow region was modelled by Prandtl-Von Karman which is given by Equation (3) (Mirko, 2012; E.J. & Franzini, 2002):

$$f = -2\log\left(\frac{2.51}{R_e\sqrt{f}}\right)^{-2} \qquad \textbf{eqn 3}$$

The Colebrook-White formula shown in Equation (4) is a result of an experimental trial by Colebrook that combines the dependence of the friction factor above with Reynold's Number and pipe relative roughness (de Souza Mendes, 2024).

$$\frac{1}{\sqrt{f}} = -2\log\left[\left(\frac{\varepsilon}{3.7d}\right) + \frac{2.51}{R_e\sqrt{f}}\right] \qquad \textbf{eqn 4}$$

In Equation (4) f is the friction factor, ε is the pipe surface roughness, usually measured in mm in SI units, d is the pipe internal diameter and R_e is the dimensionless Reynold's Number. The Colebrook-White Equation is implicit in terms of the friction factor as this factor exists on both sides of the equation. The application of Colebrook-White Equation and the Moody Diagram (described below) which depended on it has been questioned by researchers (Flack, 2018) in the transitional flow region which asymptotically approaches the hydraulically smooth region at lower Reynold's Number bordering on laminar flow and the fully rough regions at the right bordering with turbulent flow region. Research has shown that the shape of the friction factor is related to the size of eddies shed by the roughness elements for which characterisation by equivalent sand grains assumed by Nikuradse is not adequate. The difference in the roughness of pipes with that of the equivalent sand used by Nikuradse is the cause of the discrepancy (Flack & Schultz, 2010). This indicates the need for further research for modeling realistic roughness in the transitional region. However, it should be noted that this fact does not invalidate the use of the Colebrook-White Equation or methods that depend on it for the estimation of the friction factor including graphical methods such as the method proposed in this paper. Despite, this shortcoming, the Colebrook-White Equation is nonetheless is still being used as a good approximation of the friction factor in the broader range of the transition and turbulent flow regions using either numerical techniques or through graphical procedures such as the Moody Diagram or the graphical method proposed in this study.

The Moody Diagram

The Moody Diagram (1944) was developed as a graphical solution of the Colebrook-White Equation in dimensionless form and has been commonly used and referenced as a useful tool for determining friction factors as a function of Reynold's Number and pipe relative roughness values. The graphical plot of friction factor with Reynold's Number and relative roughness of pipes is shown in Figure 1 in which a discrete number of curves have been plotted to estimate the friction factor each of which corresponds to a particular relative roughness value of pipes. The curves have slopes for Reynold's Number up to 10^6 after which they tend to flatten out to a horizontal line showing little variation of friction factor with Reynold's Number. This reality is also apparent from the Colebrook-White Equation in which the friction factor term carrying the Reynold's Number approaches a negligibly small number at high Reynold's Number values that correspond to the highly turbulent regions of flow.

The Moody Diagram covers four different flow regions with the explicit formulae given by Equations (5-8) (Bengtson, 2024):

$$\text{Laminar flow}: f = \frac{64}{R_e} \qquad \textbf{eqn 5}$$

$$\text{Smooth pipe Turbulent flow}: f = \frac{0.316}{R_e^{1/4}} \qquad \textbf{eqn 6}$$

$$\text{Transition region}: f = \left\{ -2\log_{10}\left[\frac{\varepsilon/D}{3.7} + \frac{2.51}{R_e\left(f^{1/2}\right)}\right]\right\}^{-2} \qquad \textbf{eqn 7}$$

$$\text{Completely turbulent flow}: f = \left[1.14 + 2\log_{10}\left(\frac{D}{\varepsilon}\right)\right]^{-2} \qquad \textbf{eqn 8}$$

The Moody Diagram broadly covers the laminar to turbulent regions of flow. It is also an essential part of the fluid mechanics course for undergraduate students and is often preferred over the implicit Colebrook-White Equation because students at the undergraduate level may not have been exposed to the rigours of numerical techniques with which the Colebrook-White Equation can be solved. In contrast, the Moody diagram is easy to explain as well as demonstrate its use to students, using practical problems that involve the calculation of head losses. It is also worth noting that the Moody Diagram uses most of the data from Colebrook-White Equation (Huang, 2022). The friction factor variation with relative roughness in the Moody diagram grows sharply in the transition region and flattens out in the fully turbulent regions. As a result, interpolation can introduce large error in the transitional regime of flow compared to the fully turbulent region. In addition, it may not be apparent to users of the Moody Diagram that the relation between friction factor and pipe relative roughness is non-linear and logarithmic for which linear interpolation gives rise to error.

Attempts at simplifying the Moody Diagram as a more linear variation of the friction with the pipe wall relative roughness was provided by Mendes (2024) in which a normalized friction factor is used that varies with the inertia forces so that, in the laminar range of flow where such forces are not dominant, the friction factor assumes a constant value of one.

For flows in the transitional and turbulent regions, the friction factor varies more or less in a linear fashion with the relative roughness and Reynold's Number values.

However, the fact remains that the graphical solution so modified has to be plotted as a discrete number of curves, each of which corresponds to a particular pipe's relative roughness value, although they appear linear when plotted in semi-log scale.

Numerical solutions for the friction factor

Further attempts at finding a more explicit numerical formulation of the Colebrook-White Equation have been made over the years since the time the Moody Diagram was developed. As a result, several formulas have been proposed that were mainly aimed at a developing a more explicit formulation of the friction factor. Table 1 provides examples of the historical development of numerical solutions for the determination of the friction factor together with the level of precision that each method achieves. As a recent example, Brick (2011) used the Lambert W-Function to reformulate the friction factor. Offor and Abi (2016) suggested an explicit non-linear regression model to estimate the friction factor with negligible error compared with the implicit Colebrook-White Equation.

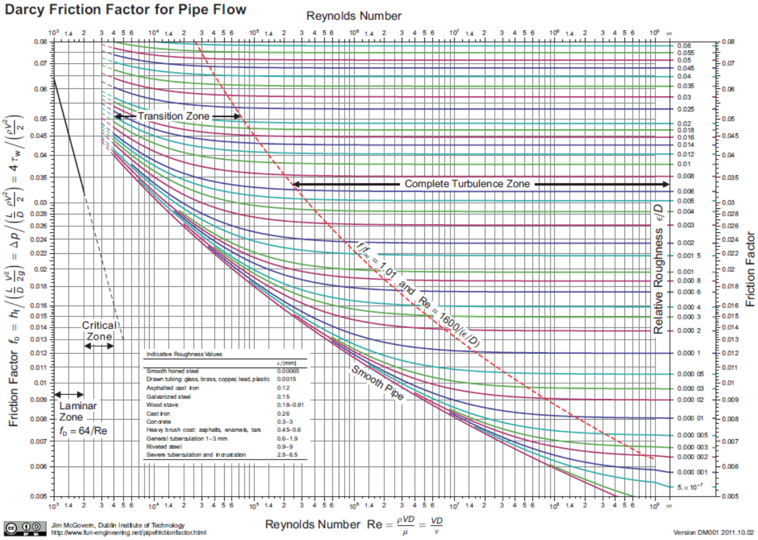

Figure 1: The Moody Diagram (Moody L. , 1944; McGovern, 2011)

Table 1: Examples of explicit numerical approximations for the determination of friction factors

Name of formula	Formula and accuracy	References
Moody (1947)	$f = 0.0055\left[1+\left(2\,X\,10^4(\varepsilon/D)+\left(\frac{10^6}{R_e}\right)\right)^{1/3}\right]$ $4000 \le R_e \le 10^8 \wedge 0 \le \left(\frac{\varepsilon}{D}\right) \le 10^{-2}$ Maximum error: ± 20%	(Mirko, 2012); (Moody L.,1947)
Altshul (1952)	$\frac{1}{\sqrt{f}} = 1.8\log\left(\frac{R_e}{R_e\left(\frac{\varepsilon}{D}\right)+7}\right)$ $4000 \le R_e \le 10^7 \wedge 0 \le \left(\frac{\varepsilon}{D}\right) \le 10^{-2}$ Maximum error: 100%	(Flack & Schultz, 2010) (Genic et al., 2011) (Mustafa, Oguz, & Mustafa, 2014) (Round, 1980) (Nekrasov, 1968)

Wood (1966)	$f=0.53\left(\frac{\varepsilon}{D}\right)+0.094\left(\frac{\varepsilon}{D}\right)^{0.225}+88\left(\frac{\varepsilon}{D}\right)^{0.44}R_e^{-1.62\left(\frac{\varepsilon}{D}\right)^{0.134}}$ $R_e>4000 \wedge 0\leq\left(\frac{\varepsilon}{D}\right)\leq 5 * 10^{-2}$ Maximum error: 28.23%	(Zeghadnia, Computation of the pressurized turbulent flow in circular pipe, 2007) (Wood, 1968)
Eck (1976)	$\frac{1}{\sqrt{f}}\cong -2\log\left(\frac{\varepsilon}{3.715\,D}+\frac{15}{R_e}\right)$ Recommended region was not specified Maximum error: 10.7%	(Eck, 1973) (Samadianfard, 2012)
Swamee and Jain (1976)	$\frac{1}{\sqrt{f}}\cong -2\log\left(\frac{\varepsilon}{3.715\,D}+\left(\frac{6.943}{R_e}\right)^{0.9}\right)$ $5000\leq R_e\leq 10^7 \wedge 4\,X\,10^{-5}\leq\left(\frac{\varepsilon}{D}\right)\leq 5\,X\,10^{-2}$ Widely used including in EPANET software Maximum error: 2.83%	(Grigson, 1984) (Jain, 1976)
Haaland (1983)	$\frac{1}{\sqrt{f}}=-1.8\log\left(\left(\frac{\varepsilon}{37\,D}\right)^{1.11}+\left(\frac{6.9}{R_e}\right)\right)$ $4000\leq R_e\leq 10^8 \wedge 10^{-6}\leq\left(\frac{\varepsilon}{D}\right)\leq 5\,X\,10^{-2}$ Maximum error: 1.41%	(Haaland, 1983)
Brkić Dejan (2011)	$\frac{1}{\sqrt{f}}\cong -2\log\left(\frac{2.18\,X\,S}{R_e}+\frac{\varepsilon}{37\,D}\right)$ $S=\ln\left(1+0.458\,R_e\right)\left(1-\frac{\ln\left(1+\ln\left(1+0.458\,R_e\right)\right)}{2+\ln\left(1+0.458\,R_e\right)}\right)$ $2300\leq R_e\leq 10^8 \wedge 0\leq\left(\frac{\varepsilon}{D}\right)\leq 5\,X\,10^{-2}$ Uses different solutions of the Lambert W-function Maximum error: 3.46%	(Brkic, 2011)

Another explicit formulation for estimating the friction factor has been provided by Vatankhah (2018) which provides a prediction of the friction factor that is closest to that proposed by the Colebrook-White Equation. The range of applicability of the numerical approximation is defined together with the formula which may not be as broad as the one suggested by the Colebrook-White Equation. Genic and Jasimovich (2019) suggested, as a result, that three different formulae should be used to address the laminar, transitional and fully turbulent flow regions

The proposed graphical procedure discussed in this paper has the objective of using a single universal curve for the estimation of friction factor and one that is applicable over the broader range of flow, covering the transition as well as turbulent flow regions and in which the Colebrook-White Equation is applicable. The procedure, in contrast with the Moody Diagram which is based on a single curve that is applicable over a continuum of pipe relative roughness and Reynolds' Number values and therefore eliminates the need for drawing several discrete curves as well as the need to interpolate the friction factor for pipe roughness values that do not have curves drawn for them. In addition, the proposed graphical procedure is intended to achieve a significant improvement in the precision of estimating the friction factor, one that places it above the very high estimation precision level normally considered as a very good estimation for a graphical procedure.

Material and Methods

The procedure for developing and using a single universal dimensionless curve that provides a graphical solution of the friction factor is presented below. Starting with the Colebrook-White Equation given earlier by Equation (4):

$$\frac{1}{\sqrt{f}}=-2\log\left[\left(\frac{\varepsilon}{3.7\,d}\right)+\frac{2.51}{R_e\sqrt{f}}\right] \qquad \textbf{eqn 4}$$

As can be shown from Equation (4), the friction factor is present on both sides of Equation (4) and thus it exists in implicit form. Rearranging Equation (4) by dividing each side by -2 gives:

$$\left(\frac{-1}{2}\right)\frac{1}{\sqrt{f}}=\log\left[\left(\frac{\varepsilon}{3.7\,d}\right)+\frac{2.51}{R_e\sqrt{f}}\right] \qquad \textbf{eqn 9}$$

Raising each side of Equation (9) to the power of 10 gives:

$$10^{\left(\frac{-1}{2}\right)\frac{1}{\sqrt{f}}}=\frac{1}{10^{\frac{1}{2\sqrt{f}}}}=\left[\left(\frac{\varepsilon}{3.7\,d}\right)+\frac{2.51}{R_e\sqrt{f}}\right] \qquad \textbf{eqn 10}$$

Multiplying each side of Equation (10) by $(f)^{1/2}/2.51$ gives:

$$\left[\frac{1}{R_e}+\frac{1}{9.287}\left(\frac{\varepsilon}{d}\right)\sqrt{f}\right]=\frac{1}{2.51}\left(\frac{\sqrt{f}}{10^{\frac{1}{2\sqrt{f}}}}\right) \qquad \textbf{eqn 11}$$

In order to facilitate plotting of a single friction factor curve as well as facilitating the determination of friction factor using this proposed graphical procedure, it is needed to define two numbers designated as M and N that will be used to transform Equation (11) through multiplication and subtraction. The ranges of variation of the numbers M and N are defined as follows:

$$M:\ -4 \leq M \leq 4\ ;\ N:\ N \geq 1 \qquad (12)$$

Transformation of Equation (11) using these two variables M and N will first be carried out by multiplying both sides of Equation (11) by 10^{M+5}:

$$\left[\frac{10^{M+5}}{R_e}+\frac{10^{M+5}}{9.287}\left(\frac{\varepsilon}{d}\right)\sqrt{f}\right]=\frac{10^{M+5}}{2.51}\left(\frac{\sqrt{f}}{10^{\frac{1}{2\sqrt{f}}}}\right) \qquad \textbf{eqn 13}$$

The second transformation is carried out by subtracting further 10^{M+N} $(f)^{1/2}$ from both sides of Equation (13) giving:

$$\left[\frac{10^{M+5}}{R_e}+\left(\frac{10^{M+5}}{9.287}\left(\frac{\varepsilon}{d}\right)-10^{M+N}\right)\sqrt{f}\right]=\frac{10^{M+5}}{2.51}\left(\frac{\sqrt{f}}{10^{\frac{1}{2\sqrt{f}}}}\right)-10^{M+N}\sqrt{f} \qquad \textbf{eqn 14}$$

The N- value in Equation (14) is generally greater than one as stated in Equation (12). Defining further a slope term η given by Equation (15) as follows:

$$\eta\left(\frac{\varepsilon}{d},M,N\right)=\frac{10^{M+5}}{9.287}\left(\frac{\varepsilon}{d}\right)-10^{M+N} \qquad \textbf{eqn 15}$$

Equation (14) now becomes:

$$\left[\frac{10^{M+5}}{R_e}+\eta\sqrt{f}\right]=\frac{10^{M+5}}{2.51}\left(\frac{\sqrt{f}}{10^{\frac{1}{2\sqrt{f}}}}\right)-10^{M+N}\sqrt{f} \qquad \textbf{eqn 16}$$

Equation (16) will be used for developing a graph. For the graphical plotting purposes, a function ϕ(f, M, N) is defined and is given by Equation (17):

$$\phi(f,M,N)=\left[\frac{10^{M+5}}{R_e}+\eta\left(\frac{\varepsilon}{d},M,N\right)\sqrt{f}\right]=\frac{10^{M+5}}{2.51}\left(\frac{\sqrt{f}}{10^{\frac{1}{2\sqrt{f}}}}\right)-10^{M+N}\sqrt{f} \qquad \textbf{eqn 17}$$

A graphical plot is now made through Equation (17) using $f^{1/2}$ as the dependent variable on the y-axis and ϕ(f, M, N) as the independent variable on the x-axis which is shown in Figure 2. This graph of ϕ(f, M.N) vs $f^{1/2}$ has a slope which is equal to η(ε/d, M, N) given by Equation (15). For a given Reynold's Number, calculation of the slope of the graph ϕ(f, M, N) gives:

$$slope=\frac{\phi(f_2,M,N)-\phi(f_1,M,N)}{\sqrt{f_2}-\sqrt{f_1}} \qquad \textbf{eqn 18}$$

$$\frac{\left(\frac{10^{M+5}}{R_e}+\eta\left(\frac{\varepsilon}{d},M,N\right)\sqrt{f_2}\right)-\left(\frac{10^{M+5}}{R_e}+\eta\left(\frac{\varepsilon}{d},M,N\right)\sqrt{f_{\square}}\right)}{\sqrt{f_2}-\sqrt{f_1}} \qquad \textbf{eqn 19}$$

$$\frac{\eta\left(\frac{\varepsilon}{d},M,N\right)\sqrt{f_2}-\eta\left(\frac{\varepsilon}{d},M,N\right)\sqrt{f_{\square}}}{\sqrt{f_2}-\sqrt{f_1}}=\eta\left(\frac{\varepsilon}{d},M,N\right) \qquad \textbf{eqn 20}$$

Therefore, the η(ε/d, M, N) variable defined above which is a function of the relative roughness ε/d is simply the slope of the ϕ(f, M, N) vs f ^{½} graph. Extending the slope line to the horizontal axis will give the ϕ-intercept, which is given by Equation (21):

$$\phi(0,M,N)=\frac{10^{M+5}}{R_e} \qquad \textbf{eqn 21}$$

Determination of optimum values of M and N (M=0 and N= 1.5)

The above formulation of graphical procedure enables the selection of suitable values of M and N that can apply to the broader ranges of Reynold's Numbers and relative roughness values with maximum efficiency or minimum error. After experimenting with different M and N values, the authors found out that the choice of M =0 and N=1.5 values gives an optimum solution with small margin of error (mostly less than 1%) to the broader ranges of Reynold's Number and relative roughness values and covering the entire range of the transition and turbulent flows and in which the Colebrook-White Equation is applied. However, this choice that is broadly applicable is not necessarily unique in that other choices still are possible. That is why a general formulation of the graphical formula given by Equation (17) is provided in order to facilitate further research and flexibility by users of the graph in future to address specific regions or provide a better precision. Furthermore, it is also possible to develop specific curves through the choice of M and N values that address particular regimes of Reynold's Number and relative roughness values for specific application purposes by future users of this proposed graphical procedure.

With this choice of M=0 and N=1.5 values, the ϕ(f, M, N) and η (ε/d, M, N) expressions are simplified by discarding the already fixed M, and N variables. After substituting the optimum values of M=0 and N=1.5, Equation (15) that express the slope η(ε/d) now take the form given by Equation (22):

$$\eta\left(\frac{\varepsilon}{d}\right)=\frac{10^5}{9.287}\left(\frac{\varepsilon}{d}\right)-10^{1.5} \qquad \textbf{eqn 22}$$

The ϕ-function given by Equation (17) for M=0 and N=1.5 likewise takes the form given by Equation (23):

$$\phi(f)=\left[\frac{10^5}{R_e}+\eta\left(\frac{\varepsilon}{d}\right)\sqrt{f}\right]=\frac{10^5}{2.51}\left(\frac{\sqrt{f}}{10^{\frac{1}{2\sqrt{f}}}}\right)-10^{1.5}\sqrt{f} \qquad \textbf{eqn 23}$$

In a similar manner, the ϕ-intercept given by Equation (21) takes the form given by Equation (24):

$$\phi(0)=\frac{10^5}{R_e} \qquad \textbf{eqn 24}$$

With these simplified formulas established through Equations (22-24), the graph ϕ(f) vs $f^{1/2}$ can be used to solve the problem of finding the friction factor graphically, which is based on application of the Colebrook – White Equation. For a given problem in which the Reynold's Number is given, the friction factor is found out graphically using the ϕ(f) vs $f^{1/2}$ curve as follows. First, the ϕ-intercept is located using Equation (24). Next, a line is extended to the ϕ-$f^{1/2}$ curve that is having a slope η calculated using Equation (22) and until it intersects the ϕ-$f^{1/2}$ curve. The intersection of this slope line with the ϕ-$f^{1/2}$ curve will give the friction factor as worked out by the Colebrook-White Equation. Conversely, for a problem in which the friction factor is given, the slope line with a slope value of η is extended from the graph with the given friction factor and the intersection of this slope line with the horizontal will give the ϕ-intercept (=$10^5/R_e$) from which the Reynold's Number R_e is calculated.

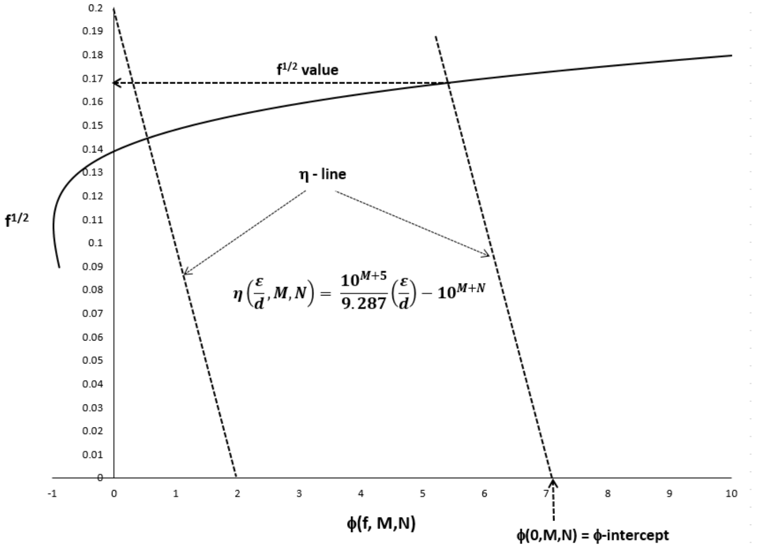

Figure 2: The ϕ (f, M, N) curve for the determination of friction factor

In summary, the following procedure is listed for the graphical method of determining the friction factor using the the ϕ(f, M) vs. $f^{1/2}$ graph. Given a problem in which the Reynold's Number R_e and the relative roughness ε/d values are given, first calculate the ϕ-intercept using Equation (24):

$$\phi(0)=\frac{10^5}{R_e} \qquad \textbf{eqn 24}$$

Next compute the slope line η(ε/d) using Equation (22)

$$\eta\left(\frac{\varepsilon}{d}\right)=\frac{10^5}{9.287}\left(\frac{\varepsilon}{d}\right)-10^{1.5} \qquad \textbf{eqn 22}$$

Draw the slope line, η, starting from the $f^{½}$ axis by choosing a suitable fi value such that this line intersects the ϕ-axis at the point which is equal to ϕ0 (fi):

$$slope=\frac{\phi_0(f_i)-0}{\sqrt{f_2}-\sqrt{0}}=\frac{\phi_0(f_i)}{\sqrt{f_2}}=\eta\left(\frac{\varepsilon}{d}\right) \qquad \textbf{eqn 25}$$

So that,

$$\phi_0(f_i)=\eta\left(\frac{\varepsilon}{d}\right)*\sqrt{f_i} \qquad \textbf{eqn 26}$$

Once the slope line,η, is established, translate this line so that the bottom of the line coincides with the ϕ-intercept, i.e., ϕ(0).

The intersection of the slope lineη and the ϕ(f) vs. $f^{1/2}$ will locate the corresponding $f^{1/2}$ which is found by drawing a horizontal line from the intersection point to the $f^{1/2}$ axis.

Calculate the corresponding friction factor using the formula: using the $f^{1/2}$ value obtained from the graph, i.e.,

$$f=\left(f^{1/2}\right)^2 \qquad \textbf{eqn 27}$$

Finally, check if this f-value is obtained correctly by substituting this f value as well as the Reynold's Number and relative roughness ε/d in the Colebrook-White formula given by Equation (4):

$$\frac{1}{\sqrt{f}}=-2\log\left[\left(\frac{\varepsilon}{3.7\,d}\right)+\frac{2.51}{R_e\sqrt{f}}\right] \qquad \textbf{eqn 4}$$

The difference between the right and left values of the friction factor in the Colebrook-White Equation above should typically be less than 1%.

RESULTS AND DISCUSSION

A demonstration of the application of the single universal graph for determining the friction factor is provided with an example given below. Water flows in a pipe that has a relative roughness value, ε/d of 0.0001 and in which the Reynold's Number, $R_e = 10^5$ is to be used. The friction factor for this flow condition is to be determined using the graphical procedure developed above.

First, the ϕ-intercept of the ϕ(f) vs. $f^{1/2}$ curve is determined using Equation (24):

$$\phi(0)=\frac{10^5}{R_e}=\frac{10^5}{10^5}=1$$

Next, the slope, η(ε/d) is calculated using Equation (22):

$$\eta\left(\frac{\varepsilon}{d}\right)=\frac{10^5}{9.287}\left(\frac{\varepsilon}{d}\right)-10^{1.5}$$

$$\eta\left(\frac{\varepsilon}{d}\right)=\frac{10^5}{9.287}(0.0001)-10^{1.5}=-30.5460$$

Choosing the f_i value of 0.2, the slope line, η(ε/d) is drawn from the $f^{1/2}$ axis to intersect the ϕ-axis at the following point:

$$\frac{\Delta f}{\Delta\phi}=\eta\left(\frac{\varepsilon}{d}\right)=-30.5460$$

$$\Delta\phi=\Delta f*\eta\left(\frac{\varepsilon}{d}\right)=0.2*(-30.5460)=-6.1092$$

Therefore, the η-line is drawn with negative slope shown in Figure 3 connecting the $f^{1/2}$ = 0.2 at the vertical f-axis with ϕ= 6.1092 at the horizontal ϕ-axis. Next, a line parallel to the η-line is moved as shown in Figure 3 until the line touches the ϕ-intercept, i.e., ϕ(0) = 1.0. This is easily produced in a graphing environment such as using the Microsoft

EXCEL graph. Otherwise, it is also possible to use a pair of set squares for sliding the η-line to the desired ϕ-intercept of ϕ=1.0.

The intersection of the η-line with the ϕ(f) curve will produce the corresponding $f^{1/2}$ value which is obtained by extending a horizontal line from the intersection point to the $f^{1/2}$ axis (Figure 3). This value is obtained as:

$f^{1/2} = 0.136$

Accordingly, the f-value is calculated as:

$$f = \left(f^{1/2}\right)^2 = (0.136)^2 = 0.018496$$

The actual value of the friction factor which is obtained by fixed point iteration using the Colebrook-White equation given by Equation (4) below gives a value of f = 0.018513866.

$$\frac{1}{\sqrt{f}} = -2\log\left[\left(\frac{\varepsilon}{3.7\,d}\right) + \frac{2.51}{R_e\sqrt{f}}\right] \qquad \textbf{eqn 4}$$

The percentage error of the graphical solution in comparison with direct application of the Colebrook-White equation is worked out as follows

$$Error(\%) = \left(\frac{0.018513866 - 0.018496}{0.018513866}\right) * 100 = 0.097\,\%$$

Therefore, the error of estimation is low, typically less than 0.1% which is acceptable for the purpose of estimating head loss based on this estimated friction factor.

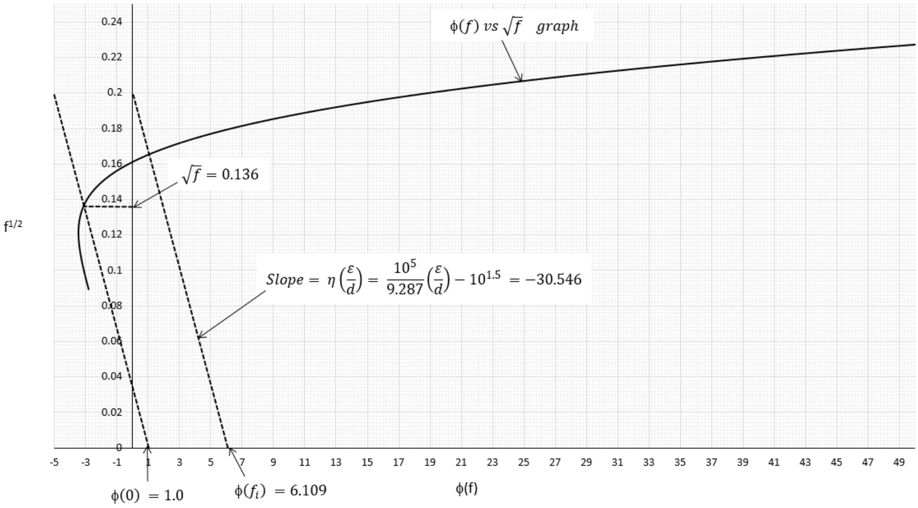

Figure 3: Solved example: Given Reynold's Number = 10^5 and Relative roughness ε/d = 0.0001, the slope η is calculated giving η= -30.546. Taking $f^{1/2} = 0.2$ makes the η-line intersect the ϕ-axis at ϕ(fi) = 6.109. Translating the η-line to the ϕ-intercept, ϕ(0) = 10^5/Re= 1.0 gives the intersection with the ϕ(f) curve at $f^{1/2} = 0.136$. This corresponds to f= 0.136^2= 0.018496

The use of the graphical procedure has been tested over a wide range of relative roughness as well as Reynold's Number values. The results are summarised in Tables 2-5. Figure 4 also displays the percentage error of estimation of the friction factor for the different ranges of Reynold's Number and pipe relative roughness values. Examination of Tables 2-5 as well as Figure 4 shows that the percentage error of estimation is mostly less than 0.1%. The mean value of the percentage error is around 0.36%. Only two of the estimations exceed 1% with a maximum percentage error of 2.3% for $\varepsilon/d = 10^{-4}$ and Reynold's Number, $R_e = 10^7$.

The error of estimation tends to increase at very high Reynold's Number values because the ϕ-intercept approaches zero and drawing the slope line from these indistinguishably small ϕ-intercept points gives f-values that are close to each other. However, this is to be expected since the friction factor tends to remain constant and does not show appreciable variation with the Reynold's Number at the highly turbulent region, that is beyond Reynold's Number, R_e of 10^7.

Overall, it can be stated with reasonable confidence that the proposed graphical method has an acceptably low error of estimation that is mostly in the 0.1% range and such percentage error is acceptably small for the purpose of estimation of head losses over pipe lengths encountered commonly in pipe distribution systems.

According to Brkic (2011), explicit numerical solutions to friction factor are classified based on the criteria of relative error as follows: Extremely accurate (≤0.14%), Very accurate (≤0.5%), moderately accurate (≤1.5%), less accurate (≤5%), non-advisable (≤25%) and extremely inaccurate (≥80%). Based on Brkic criteria, the proposed graphical method falls in the range of extremely accurate to very accurate range, which, together with being a graphical method, is acceptable to be used for the estimation of friction factor. It is also necessary to contextualise the importance of the graphical solution produced in this paper in relation to the pioneer graphical solution, i.e., Moody Diagram. Estimation of the friction factor using the Moody diagram can produce an error as high as 20%. In addition, the Moody diagram consists of a discrete number of curves and does not cover the entire continuum of relative roughness, ε/d, values. This condition requires having to guess the friction factor from the curves in the Moody Diagram when the relative roughness lies between those values for which the curves are available in the diagram. Such guessing can be subjective and likely a source of error in the estimation of friction factors.

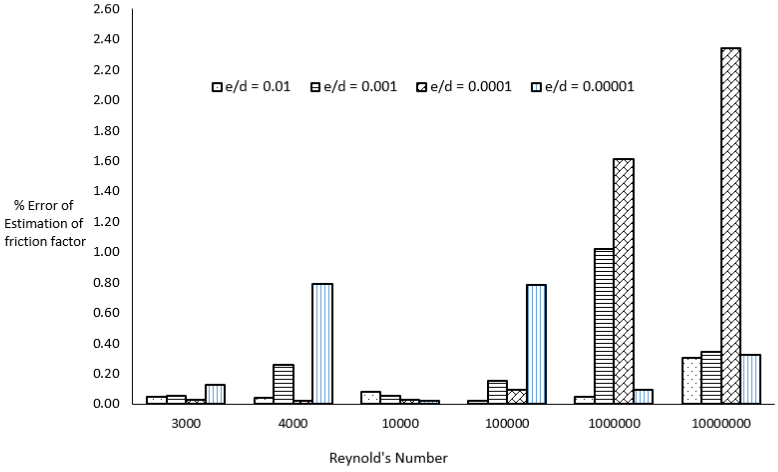

Figure 4: Percentage Error of estimation of friction factor for different ranges of pipe relative roughness and Reynold's Number values

In comparison with the Moody Diagram, the proposed graphical method has advantage of giving a low error of estimation over the wide range of relative roughness and Reynold's Number values as well as enabling estimation of the friction factor over the continuous range of relative roughness values encountered. In other words, the proposed method is based on a single curve and does away with the necessity of plotting a discrete number of curves, each of which corresponds to different values of pipe relative roughness, ε/d. The proposed method is a single curve that is universally applicable for all ranges of the relative roughness values encountered.

Another feature of the Moody Diagram is the logarithm scale with which the Reynold's Number has to be plotted, which is necessitated by the fact that the ranges of Reynold's Number encountered in the turbulent range is wide. Reading a Reynold's Number with a logarithm scale is a little difficult. The proposed method plots both the ϕ(f) as well as $f^{1/2}$ in linear (arithmetic scale) and does away with the need to use a log scale for plotting. Reading values from this graph at a linear scale is easier. The nature of the Colebrook-White Equation is such that developing graphs that use linear scales in both the x and y axes difficult. However, the proposed method adopts linear scales while accommodating the wider ranges over which both the relative roughness as well as Reynold's Number values vary. In comparison with the relative errors of estimation of friction factor by several of the earlier methods such as the ones by Moody, Haaland, Wood, Eck, Churchill and others (Vasconcello, 2020), the proposed graphical method performs better. This merit should be judged in the context of the proposed method being a graphical procedure which cannot always be as precise as the numerical formulae.

The graphical method presented in this study can be used to estimate the friction factor both manually as well as by using graphical plotting software programmes. The use of commonly available software programmes such as Microsoft EXCEL creates a suitable environment for using this graphical method. By zooming the graphs large in Microsoft EXCEL for example, it is possible to increase the accuracy of estimation of the friction factor compared with the manual method. Manual methods require using a pair of set squares for the purpose of sliding a line parallel to the slope, η(ε/d) until it intersects the ϕ-intercept, i.e., $\phi(0) = 10^5/R_e$. This is also relatively easy to perform.

With the inherent drawbacks of all formulas including the Colebrook-White Equation to accurately simulate flow conditions in all flow regimes, graphical methods should be able to adapt to the changing flow regimes at different pipe relative roughness and Reynold's Numbers. The Moody Diagram as such uses the four flow regimes to model the friction factor variation. In this context, the proposed graphical method can be adapted particularly with the modification of the Colebrook-White Equation to address and fit the data from Nikuradse's experiment at higher pipe relative roughness values. Accordingly, (Brkić & Pavel, 2018) the Colebrook-White formula given by Equation (4) can be modified with the following formula:

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{2.51}{R_e}\right) \cdot \frac{1}{\sqrt{f}} + \frac{\varepsilon}{3.71} \cdot e^{E} \qquad \textbf{eqn 28}$$

$$E = \frac{-31.13}{R_e\,\varepsilon} \cdot \frac{1}{\sqrt{f}} \qquad \textbf{eqn 29}$$

When E = 0, Equation (28) reduces to the original Colebrook-White Equation given by Equation (4). Noting that none of the currently available formulas are valid for all hydraulic regimes, the above modification of the Colebrook-White Equation can be similarly modelled into the proposed graphical solution discussed with a similar arrangement in which the ϕ-function will remain a monolithic function of the friction factor and the slope line varies with the pipe relative roughness only. The proposed graphical solution as such retains its single universal curve characteristics while improving the accuracy and range of applicability for the determination of the friction factor. Lastly, with the presence of quite a number of proposed numerical methods that explicitly determine the friction factor, often with high precision, and the presence of software programmes that merely require users to input data, it may be difficult to make a case for graphical methods. However, a graphical solution provides an alternative that is simple to carry out but not necessarily to replace such highly precise numerical procedures. Using graphs does not require having to write formula or programs that are needed by numerical solutions and can be applied by a broader range of users. It is, therefore, in this context that this paper suggests that the proposed graphical method has relevance to users interested in solving pipe flow problems involving the friction factor. In addition, graphical solution can be used as a guide for a determining a suitable starting point for numerical iteration, which will quickly converge, to the solution with such educated guesses.

Table 2: Estimation of friction factor for ε/d=0.01 using a single universal graph and percentage error by comparison with the numerical solution using the Colebrook-White equation

Reynolds Number	ε/d	ϕ-intercept	η	fi1/2	ϕ-intercept slope	$f^{1/2}$ from graph	f - estimated	f-calculated Colebrook-White	Percentage Error of Estimation
3000	0.01	33.333	76.055	0.3	22.816	0.2278	0.051893	0.051868	0.047
4000	0.01	25.000	76.055	0.3	22.816	0.2215	0.049062	0.049082	0.041
10000	0.01	10.000	76.055	0.3	22.816	0.20775	0.043160	0.043127	0.078
100000	0.01	1.000	76.055	0.3	22.816	0.1962	0.038494	0.038504	0.024
1000000	0.01	0.100	76.055	0.3	22.816	0.1948	0.037947	0.037965	0.047
10000000	0.01	0.010	76.055	0.3	22.816	0.195	0.038025	0.037910	0.304

Table 3: Estimation of friction factor for ε/d=0.001 using single universal graph and percentage error by comparison with the numerical solution using the Colebrook-White equation

Reynolds Number	ε/d	ϕ-intercept	η	$fi^{1/2}$	ϕ-intercept slope	$f^{1/2}$ from graph	f - estimated	f-calculated Colebrook-White	Percentage Error of Estimation
3000	0.001	33.333	-20.855	0.2	-4.171	0.2108	0.044437	0.044411	0.057
4000	0.001	25.000	-20.855	0.2	-4.171	0.202	0.040804	0.040910	0.260
10000	0.001	10.000	-20.855	0.2	-4.171	0.18	0.032400	0.032382	0.056
100000	0.001	1.000	-20.855	0.2	-4.171	0.1488	0.022141	0.022175	0.149
1000000	0.001	0.100	-20.855	0.2	-4.171	0.1405	0.019740	0.019943	1.019
10000000	0.001	0.010	-20.855	0.2	-4.171	0.14	0.019600	0.019667	0.341

Table 4: Estimation of friction factor for ε/d=0.0001 using single universal graph and percentage error by comparison with the numerical solution using the Colebrook-White equation

Reynolds Number	ε/d	ϕ-intercept	η	$fi^{1/2}$	ϕ-intercept slope	f1/2 from graph	f - estimated	f-calculated Colebrook-White	Percentage Error of Estimation
3000	0.0001	33.333	-30.546	0.2	-6.109	0.2088	0.043597	0.043609	0.027
4000	0.0001	25.000	-30.546	0.2	-6.109	0.2	0.040000	0.040008	0.021
10000	0.0001	10.000	-30.546	0.2	-6.109	0.1762	0.031046	0.031037	0.030
100000	0.0001	1.000	-30.546	0.2	-6.109	0.136	0.018496	0.018514	0.097
1000000	0.0001	0.100	-30.546	0.2	-6.109	0.115	0.013225	0.013441	1.610
10000000	0.0001	0.010	-30.546	0.2	-6.109	0.109	0.011881	0.012166	2.343

Table 5: Estimation of friction factor for ε/d=0.00001 using single universal graph and percentage error by comparison with the numerical solution using the Colebrook-White equation

Reynolds Number	ε/d	ϕ-intercept	η	fi1/2	ϕ-intercept slope	$f^{1/2}$ from graph	f - estimated	f-calculated Colebrook White	Percentage Error of Estimation
3000	0.00001	33.333	-31.515	0.2	-6.303	0.2085	0.043472	0.043528	0.129
4000	0.00001	25.000	-31.515	0.2	-6.303	0.199	0.039601	0.039917	0.792
10000	0.00001	10.000	-31.515	0.2	-6.303	0.1758	0.030906	0.030898	0.023
100000	0.00001	1.000	-31.515	0.2	-6.303	0.1338	0.017902	0.018044	0.783
1000000	0.00001	0.100	-31.515	0.2	-6.303	0.109	0.011881	0.011870	0.097
10000000	0.00001	0.010	-31.515	0.2	-6.303	0.095	0.009025	0.008996	0.326

CONCLUSION

Several formulae for modelling the variation of friction factor have been proposed historically over the years that are applicable in the transition and turbulent flow regions in which the friction factor is dependent on both the pipe roughness as well as Reynold's number values. While the Colebrook-White Equation addresses the broader regime of flow,including both the transition as well as turbulent flow regions, the implicit nature of the formula limits its direct application for determining the friction factor. As a result, several formulae have been proposed for the explicit determination of the friction factor. Apart from the need for an explicit form of solution, improvement over the Colebrook-White Equation in the transitional flow region where such equation is considered inadequate have been addressed by a number of the proposed numerical solutions. The Moody diagram provided a graphical solution which is for estimation of friction factor.. This paper discussed the use of a graphical method that is based on a single, universal curve for the graphical determination of friction factor over the broader range of pipe relative roughness and Reynold's Number values. The proposed method is simple to develop and apply. It is a significant improvement over the Moody Diagram in terms of its precision. Application of the proposed graphical procedure over the broader range of transition and turbulent flow regions indicated that the overall mean percentage error of estimation is 0.36% and the maximum error is 2.3%. This result places the graphical method so developed in the extremely accurate to very accurate estimation level. Unlike the Moody Diagram that requires using four different formulas for the four different flow regimes, the proposed graphical method uses a single formula based on the Colebrook-White Equation that covers the entire flow regime of the laminar, transitional and fully turbulent flow regimes. Moreover, the proposed method avoids interpolation in between the discrete number of relative roughness curves. Numerical solution, though stated in explicit form and being precise, are not universally applicable for all flow regimes. The proposed graphical method, by contrast, can be modified to fit the different flow regimes as was also discussed earlier with respect to adaptation of the Colebrook-White Equation to fit with the Nicuradse experiment addressing higher pipe relative roughness values.

Declaration of conflict of interest

The authors declare no conflict of interest.

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